

## CLAIMS

What is claimed is:

1. A method for solving a Model Predictive Control problem including the steps of:

a) forming a Large Sparse Matrix Equation, LSME, based upon the Model Predictive Control problem;

b) repeatedly, at least once per controller update, sampling commands and feedback sensors to pose a determination of actuator commands in terms of a solution of a quadratic programming problem based upon model predictive control;

c) forming a square root of  $H$ ,  $H_r$ , where  $H$  is a submatrix of the LSME that varies only once per controller update and  $H_r$  is a block triangular matrix in which each block along a diagonal is triangular;

d) forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in each of a plurality of iterations of a quadratic programming solver within each controller update, and  $LSM_{root}$  is block triangular matrix in which each block along the diagonal is triangular; and

e) completing the solution of the LSME based upon  $LSM_{root}$  in each iteration.

2. The method of claim 1 wherein  $H_r$  is calculated in a first of the plurality of iterations and is not recalculated for subsequent ones of the plurality of iterations.

3. The method of claim 2 wherein in said step c),  $H_r$  is formed without first forming  $H$ .

4. The method of claim 3 wherein said step c) is performed with a sequence of QR factorizations.
5. The method of claim 3 where  $H = H_r' * S * H_r$  and  $H_r = L W^T$  where

$$L = \begin{bmatrix} F_0 & & & & & & & & & & \\ -M_1 B_0 & I & & & & & & & & & \\ & 0 & I & -M_1 & & & & & & & \\ & & 0 & G_1 & F_1 & & & & & & \\ & & & -M_2 A_1 & -M_2 B_1 & I & & & & & \\ & & & & 0 & I & -M_2 & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 0 & G_{N-1} & F_{N-1} & & \\ & & & & & & & -M_N A_{N-1} & -M_N B_{N-1} & I & \\ & & & & & & & & 0 & I & -M_N \end{bmatrix}$$

$$S = \begin{bmatrix} I & & & & & & & & & & \\ & -I & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & -I & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & -I & & \\ & & & & & & & & & I & \end{bmatrix}$$

$$W = \begin{bmatrix} I & & & & & & & & & & \\ & M_1 & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & M_2 & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & M_N & & \\ & & & & & & & & & I & \end{bmatrix}$$

6. The method of claim 5 where the submatrices F, G, and M are produced by the recursive sequence of QR factorizations:

$$\begin{bmatrix} Q^{1/2}C_n & Q^{1/2}D_n \\ M_nA_{n-1} & M_nB_{n-1} \\ 0 & R^{1/2} \end{bmatrix} = Ortho^* \begin{bmatrix} 0 & 0 \\ M_{n-1} & 0 \\ G_{n-1} & F_{n-1} \end{bmatrix}$$

for various future controller update time points in the prediction horizon.

7. The method of claim 1 wherein the LSMroot is formed in said step d) without first forming the LSM.
8. The method of claim 7 wherein said step d) is performed by solving a normal or generalized QR factorization.
9. The method of claim 7 wherein said step d) is performed by solving a generalized QR factorization customized for an interior point method.
10. The method of Claim 9 where the generalized QR factorization is

$$\begin{bmatrix} 0 \\ P \end{bmatrix} = U \cdot Pin, \text{ where } Pin = \begin{bmatrix} T^{-T} J \\ L \end{bmatrix}, U^T \begin{bmatrix} I \\ S \end{bmatrix} U = \begin{bmatrix} I \\ S \end{bmatrix}$$

where Pin and S are inputs and triangular P and, possibly, U are outputs, and where

$$LSM_{root} \equiv \begin{bmatrix} T & -T^{-T} J \\ 0 & P \end{bmatrix} \cdot \begin{bmatrix} I \\ W^{-T} \end{bmatrix} \cdot \text{and } Sr=I.$$

11. The method of claim 7 wherein said step d) is performed by solving a generalized QR factorization customized for an active set method.
12. The method of Claim 11 where the generalized QR factorization is

$$S_R^{1/2} P = S^{1/2} U \cdot Pin, \text{ where } Pin = L^{-T} J^T, U^T S \cdot U = S$$

where Pin and S are inputs and P and Sr, and possibly, U are outputs and where

$$LSM_{root} = \begin{bmatrix} P & 0 \\ SL^{-T}J^T & L \end{bmatrix} \begin{bmatrix} I \\ W^{-T} \end{bmatrix}$$

13. The method of Claim 12 where the generalized QR factorization is accomplished via a standard Householder transform based QR algorithm that is modified by replacing the sequence of Householder transforms

$$U_i = \left( I - 2 \frac{v \cdot v^T}{v^T \cdot v} \right), \text{ which are normally used with a corresponding sequence}$$

of generalized transformations  $U_i = \left( I - 2 \frac{v \cdot v^T S}{v^T S \cdot v} \right)$ . U is unneeded but, if desired, is the cumulative product of the  $U_i$ .

14. The method of claim 13 where the result of generalized QR factorization, which is a triangular matrix consisting of rows containing either purely real numbers or purely imaginary numbers, is further factored into the product of a diagonal matrix,  $S_R^{1/2}$ , which has either 1 or the square root of  $-1$  on the diagonal, and a purely real triangular matrix, and wherein the square of this diagonal matrix,  $S_r$ , will then have either 1 or  $-1$  on the diagonal, and  $S_r$  and  $P$  are purely real numbers.

15. The method of claim 11 further including the step of factoring the LSM to use only real numbers as:

$$LSM = LSM_{root}^T \begin{bmatrix} -S_R & 0 \\ 0 & S \end{bmatrix} LSM_{root}$$

where S and SR are constant diagonal matrices with only 1 or -1 entries along the diagonal, and where LSMroot is a block triangular matrix where each diagonal submatrix is also triangular.

16. The method of claim 1 wherein the LSME has the form:

$$\begin{bmatrix} -T^T T & J \\ J^T & H \end{bmatrix} \begin{bmatrix} m \\ z \end{bmatrix} = \begin{bmatrix} K \\ f \end{bmatrix}$$

where z is a vector containing states, controls and state equation adjoint variables for each time point, grouped by time; m includes adjoint variables for inequality constraints, grouped by time; f and K are vectors; H and J are banded matrices; and T is a diagonal matrix or zero, depending on the quadratic program algorithm selected.

17. A method for controlling a system including the steps of:
- a) Receiving a plurality of sensor signals indicating current conditions of the system;
  - b) Receiving a plurality of commands;
  - c) Determining a desired dynamic response of the system based upon the commands;
  - d) in each of a plurality of time steps, formulating a problem of achieving a desired dynamic response for a window spanning multiple time steps as a solution to a quadratic program problem using methods of model predictive control and the sensor signals;
  - e) solving the quadratic programming problem in each time step using an iterative algorithm in which a Large Sparse Matrix Equation, LSME, is formed based upon the model predictive control problem and a quadratic programming algorithm;
  - f) in a first iteration of the iterative algorithm, forming a square root of  $H$ ,  $H_r$ ;
  - g) in iterations including the first iteration and iterations subsequent to the first iteration of the iterative algorithm, forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in a quadratic programming solver; and
  - h) in iterations including the first iteration and iterations subsequent to the first iteration of the iterative algorithm, completing the solution of the LSME based upon  $LSM_{root}$ .

18. A model predictive control system for controlling a plant comprising:
- a plurality of sensors indicating a current state of the system;
  - a desired trajectory generator for creating a desired dynamic response based upon commands;
  - starting with a current state of the system, a quadratic programming module formulating a problem of achieving the desired dynamic response for a window spanning multiple time steps as a solution to a quadratic program problem using methods of model predictive control;
  - a quadratic programming solver solving the quadratic programming problem in each time step using an iterative algorithm in which a Large Sparse Matrix Equation, LSME, is formed based upon the model predictive control problem and a quadratic programming algorithm, the solver forming a square root of  $H$ ,  $H_r$ , in a first iteration of the iterative algorithm, the solver forming a square root of a Large Sparse Matrix of the LSME,  $LSM_{root}$ , using  $H_r$  in a quadratic programming solver in iterations including the first iteration and a plurality of iterations subsequent to the first iteration of the iterative algorithm, the solver completing the solution of the LSME based upon  $LSM_{root}$  in iterations including the first iteration and the plurality of iterations subsequent to the first iteration of the iterative algorithm.
19. The system of claim 18 wherein the solver forms  $H_r$  in the first iteration and does not recalculate  $H_r$  in the plurality of iterations subsequent to the first iteration of the iterative algorithm.
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20. The system of claim 19 wherein the solver forms Hr without first forming H.

21. The system of claim 20 wherein  $H = H_r' * S * H_r$  and  $H_r = L W^T$  where

$$L = \begin{bmatrix} F_0 & & & & & & & & & & \\ -M_1 B_0 & I & & & & & & & & & \\ 0 & & I & -M_1 & & & & & & & \\ & & 0 & G_1 & F_1 & & & & & & \\ & & & -M_2 A_1 & -M_2 B_1 & I & & & & & \\ & & & & 0 & I & -M_2 & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & 0 & G_{N-1} & F_{N-1} & \\ & & & & & & & & -M_N A_{N-1} & -M_N B_{N-1} & I \\ & & & & & & & & & 0 & I & -M_N \end{bmatrix}$$

$$S = \begin{bmatrix} I & & & & & & & & & & \\ & -I & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & -I & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & -I & & \\ & & & & & & & & & I & \end{bmatrix}$$

$$W = \begin{bmatrix} I & & & & & & & & & & \\ & M_1 & & & & & & & & & \\ & & I & & & & & & & & \\ & & & I & & & & & & & \\ & & & & M_2 & & & & & & \\ & & & & & I & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & I & & & \\ & & & & & & & & M_N & & \\ & & & & & & & & & I & \end{bmatrix}$$

22. The system of claim 21 where the submatrices F, G, and M are produced by the recursive sequence of QR factorizations:

$$\begin{bmatrix} Q^{1/2} C_n & Q^{1/2} D_n \\ M_n A_{n-1} & M_n B_{n-1} \\ 0 & R^{1/2} \end{bmatrix} = Ortho^* \begin{bmatrix} 0 & 0 \\ M_{n-1} & 0 \\ G_{n-1} & F_{n-1} \end{bmatrix}$$

for various future controller update time points in the prediction horizon.



23. The system of claim 18 wherein the solver forms LSMroot without first forming the LSM.

24. The system of claim 23 wherein the solver forms the LSMroot by solving a normal or generalized QR factorization.

25. The system of claim 23 wherein the solver forms the LSMroot by solving a generalized QR factorization customized for an interior point method.

26. The system of claim 25 where the generalized QR factorization is

$$\begin{bmatrix} 0 \\ P \end{bmatrix} = U \cdot Pin, \text{ where } Pin = \begin{bmatrix} T^{-T} J \\ L \end{bmatrix}, U^T \begin{bmatrix} I \\ S \end{bmatrix} U = \begin{bmatrix} I \\ S \end{bmatrix}$$

where Pin and S are inputs and triangular P and, possibly, U are outputs, and where

$$LSM_{root} \equiv \begin{bmatrix} T & -T^{-T} J \\ 0 & P \end{bmatrix} \cdot \begin{bmatrix} I \\ W^{-T} \end{bmatrix} \cdot \text{and } Sr=I.$$

27. The system of claim 23 wherein the solver forms LSMroot by solving a generalized QR factorization customized for an active set method.

28. The system of Claim 27 where the generalized QR factorization is

$$S_R^{1/2} P = S^{1/2} U \cdot Pin, \text{ where } Pin = L^{-T} J^T, U^T S \cdot U = S$$

where Pin and S are inputs and P and Sr, and possibly, U are outputs and where

$$LSM_{root} = \begin{bmatrix} P & 0 \\ SL^{-T}J^T & L \end{bmatrix} \begin{bmatrix} I \\ W^{-T} \end{bmatrix}$$

29. The system of Claim 28 where the generalized QR factorization is accomplished via a standard Householder transform based QR algorithm that is modified by replacing the sequence of Householder transforms

$$U_i = \left( I - 2 \frac{v \cdot v^T}{v^T \cdot v} \right), \text{ which are normally used with a corresponding sequence}$$

$$\text{of generalized transformations } U_i = \left( I - 2 \frac{v \cdot v^T S}{v^T S \cdot v} \right). \text{ U is unneeded but, if}$$

desired, is the cumulative product of the  $U_i$ .

30. The system of claim 29 where the result of generalized QR factorization, which is a triangular matrix consisting of rows containing either purely real numbers or purely imaginary numbers, is further factored into the product of a diagonal matrix,  $S_R^{1/2}$ , which has either 1 or the square root of  $-1$  on the diagonal, and a purely real triangular matrix and wherein a square of this diagonal matrix,  $S_r$ , will then have either 1 or  $-1$  on the diagonal, and  $S_r$  and  $P$  are purely real numbers.

31. The system of claim 27 wherein the solver factors the LSM to use only real numbers as:

$$LSM = LSM_{root}^T \begin{bmatrix} -S_R & 0 \\ 0 & S \end{bmatrix} LSM_{root}$$

where S and SR are constant diagonal matrices with only 1 or -1 entries along the diagonal, and where LSMroot is a block triangular matrix where each diagonal submatrix is also triangular.

32. The system of claim 18 wherein the LSME has the form:

$$\begin{bmatrix} -T^T T & J \\ J^T & H \end{bmatrix} \begin{bmatrix} m \\ z \end{bmatrix} = \begin{bmatrix} K \\ f \end{bmatrix}$$

where z is a vector containing states, controls and state equation adjoint variables for each time point, grouped by time; m includes adjoint variables for inequality constraints, grouped by time; f and K are vectors; H and J are banded matrices; and T is a diagonal matrix or zero, depending on the quadratic program algorithm selected.